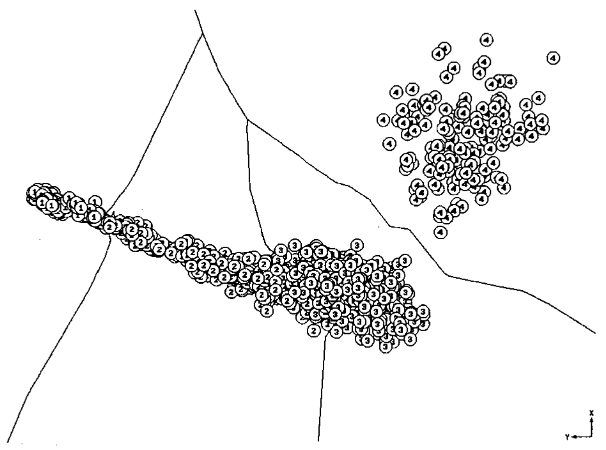

Fig. 4 Projection of four-dimensional FFT feature vectors.

The stall has been correctly identified every time. Initially, the classification results in the remaining compressor operating classes are unsatisfactory. The errors are borne in the boundaries forming the different classes. We developed a visualization tool for the direct display of high-dimensional feature vectors in a two-dimensional plane. A planar mapping of the four-dimensional FFT coefficients feature vectors is given in Fig. 4, each of the vectors is given in one circle. Because the change of the compressor operating states is continuous until stall inception and the boundaries in Fig. 1 are to some extent empiric, it is hardly possible and also not necessarily desirable, to have a 100% classification result in all boundaries except at the stall line.

V. Conclusions

This work provides a viable and robust method for estimating compressor operating states including stall. Current work focuses on the real-time implementation of our operating point estimation in a rapid prototyping hardware environment. For the classification unit, an application specific integrated circuit (ASIC) has been fabricated. Extension of this work to other compressor types are being conducted at a transonic compressor test bed at Darmstadt University and at an industrial gas turbine.

Acknowledgment

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Shockless Transition from Supersonic to Subsonic Flow

E. William Beans*
University of Toledo, Toledo, Ohio 43606

Nomenclature

A = duct area
D = duct diameter
f = friction factor
k = ratio of specific heats
M = Mach number
p = pressure
p_0 = total pressure
R = gas constant
s = entropy
T = temperature
T_0 = total temperature
x = distance along the duct
α = constant defined by Eq. (2)
β = constant defined by Eq. (3)
ϕ = duct angle

Subscript
$*$ = value at the sonic point

Introduction

A NUMERICAL solution for transition from supersonic to subsonic flow without passing through a shock was presented by Beans.[1] In this Note it was stated that the numerical solution agrees with experimental results observed by Rothe.[2] It was further stated by Beans[1] that a number of shockless solutions are possible. The numerical method used by Beans[1] was a generalized one-dimensional flow that is described by Shapiro.[3]

The object of this Note is to show that shockless transition from supersonic to subsonic flow is theoretically possible. This Note will also show that by proper selection of the independent variable a series of shockless transition function exists. Generalized one-dimensional flow will be used to show these results.

Analysis

The basic differential equation for generalized one-dimensional flow is presented in Eq. (1):

$$\frac{dM^2}{M^2} = \left[\frac{1 + (k - 1)M^2/2}{(1 - M^2)}\right]\left[\frac{-2\,dA}{A} + (1 + kM^2)\frac{dT_0}{T_0} + kM^2 4\frac{f\,dx}{D}\right] \tag{1}$$

The above equation assumes a constant k. The equation is limited to independent variations in A, T_0, and f. This is done for the sake of brevity. With known relationships for area, total temperature, and friction, the nonlinear differential equation can be solved numerically. This is the method used by Beans.[1]

The mathematical difficulty in continuous transition from supersonic to subsonic flow or vice versa is the singularity that exists at $M = 1$ [see Eq. (1)]. Shapiro[3] handles the singularity by using L'Hospital's Rule. The approach used in this Note will be to remove the singularity.

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*Professor Emeritus, Department of Mechanical Engineering.

The analysis will be limited to circular ducts with constant f. The following relationships between f, T_0, and D are assumed:

$$\frac{fk\,\mathrm{d}x}{D} = \frac{\alpha\,\mathrm{d}D}{D} \tag{2}$$

$$\frac{dT_0}{T_0} = \frac{\beta\,\mathrm{d}D}{D} \tag{3}$$

where α and β are general constants. Equations (2) and (3) are now substituted into Eq. (1).

The singularity is removed by setting the independent variable portion of Eq. (1) equal to the singularity. Or

$$(1 - M^2) = (-4 + \beta + k\beta M^2 + 4\alpha M^2) = -(4 - \beta) + (k\beta + 4\alpha)M^2$$

Therefore, $1 = -4 + \beta = -(k\beta + 4\alpha)$. Hence, the required relationship between β and α is

$$\beta = 4(1 - \alpha)/(k + 1) \tag{4}$$

It should be noted that to remove the singularity, a relationship between two of the three independent variables is required. All three independent variables can be used. This is the case examined by Beans in Ref. 1. Therefore, a smooth transition requires a unique relationship between area change, heat transfer, and frictional effects.

The singularity can be removed with the single variable of area change in isentropic flow ($\mathrm{d}T_0 = 0$ and $f = 0$). In this case, the area change $\mathrm{d}A$ must equal 0 at the same point where $M = 1$. The duct, therefore, must have a throat. The problem with this isentropic single variable change is the downstream flow can just as readily be supersonic or subsonic. A two or three variable change provides direction or stability to the supersonic to subsonic transition.

Substituting Eq. (4) into Eq. (1), one has

$$\frac{\mathrm{d}M^2}{M^2[1 + (k - 1)M^2/2]} = \frac{-4(k + \alpha)}{(k + 1)}\frac{\mathrm{d}D}{D}$$

Integrating from $M = 1$ and $A = A^*$ to $M = M$ and $A = A$, the relationship between area and Mach number for shockless transition is

$$\frac{A}{A^*} = \left[\frac{1 + (k - 2)M^2/2}{M^2(k + 1)/2}\right]^{[(k+1)/2(k+\alpha)]} \tag{5}$$

Integrating Eqs. (2) and (3) and substituting Eq. (5) into the result, the other independent relationships are

$$\frac{T_0}{T_0^*} = \left[\frac{1 + (k - 1)M^2/2}{M^2(k + 1)/2}\right]^{[(1-\alpha)/(k+\alpha)]} \tag{6}$$

$$fkx/D^* = \alpha\left\{\left[\frac{1 + (k - 1)M^2/2}{M^2(k + 1)/2}\right]^{[(k+1)/4(k+\alpha)]} - 1\right\} \tag{7}$$

The values for area and total temperature at $M = 1$, which are A^* and T_0^*, have been selected as the reference values. The value of x has arbitrarily been set equal to 0 at Mach 1.

From the continuity equation, perfect gas relation, and the definition of the Mach number, the auxiliary equations are

$$\frac{T}{T^*} = \left[\frac{1 + (k - 1)M^2/2}{(k + 1)/2}\right]^{[(k+2\alpha-1)/(k+\alpha)]} \Big/ M\,\frac{2(1 - \alpha)}{k + \alpha} \tag{8}$$

$$p/p^* = [(k + 1)/2]/[1 + (k - 1)M^2/2] \tag{9}$$

$$p_0/p_0^* = [1 + (k - 1)M^2/2]/[(k + 1)/2]^{1/(k-1)} \tag{10}$$

$$\frac{s - s^*}{R} = \left[\frac{k(\alpha - 1)}{(k + \alpha)(k - 1)}\right]\ln(M^2) - \left[\frac{\alpha(k + 1)}{(k + \alpha)(k - 1)}\right]\ln\left[\frac{1 + (k - 1)M^2/2}{(k + 1)/2}\right] \tag{11}$$

Notice that the variations in static p and p_0 are independent of the constants α and β.

One can see from Eqs. (7), (6), and (5) that the following special cases occur when 1) $\alpha = 0$, frictionless flow with area and total temperature change; 2) $\alpha = 1$, adiabatic flow with area change and friction; and 3) $\alpha \to \infty$, constant area flow with total temperature change and friction.

The bracketed Mach term in Eqs. (5–7) varies continuously from less than 1 to greater than 1 as the Mach number varies from supersonic to subsonic. One must also note that f must always be positive. Therefore, the direction of the transition of the flow is controlled by the exponents of these equations, which is a function of the value of α. There are three regions of interest in the value of α.

When α is greater than 0, the exponents in Eqs. (5) and (7) are positive. Since f must be positive, the flow can only transition from supersonic to subsonic [see Eq. (7)], and the duct must be divergent [see Eq. (5)]. No throat or shock is necessary for this. Continuous transition from subsonic to supersonic is not possible without a throat.[1]

When α is between 0 and 1, the flow is heated or total temperature is increased. When α is greater than 1, the flow is cooled [see Eq. (6)]. When $\alpha = 1$, the flow is adiabatic, see listing above.

As stated, the flow is frictionless when $\alpha = 0$. Therefore, transition from supersonic to subsonic is possible in a divergent duct with heating. Transition from subsonic to supersonic is also possible in a convergent duct with cooling.

When α is between 0 and $-k$, the exponents in the three equations are all positive. The requirement that friction be positive results in transition from subsonic to supersonic as being the only possible solution. The duct must be convergent in this case and it does not have a throat. To obtain this subsonic to supersonic transition the total temperature must decrease.

A singularity exists at $\alpha = -k$. Equations (5–7) have a single value for $M > 1$ and another single value for $M < 1$, regardless of the value of the Mach number in these ranges. The equations are discontinuous at $M = 1$. The values for the equations in these ranges are shown in Table 1.

For $\alpha < -k$, only supersonic to subsonic is possible. In this range, the duct is convergent and the total temperature decreases with length.

Examples

Figure 1 presents the variation in total temperature, friction, and area with Mach number for $\alpha = 0.5$. One can see that the three variables change continuously through the sonic point. Since the difference in the friction variable (fkx/D^*) must be positive, (+) − (−), the direction of change can only be from supersonic to subsonic. Hence, the area is divergent and the total temperature is increased.

Table 1 Singularity point values at $\alpha = -k$

	$M < 1$	$M = 1$	$M > 1$
A/A^*	∞	1	0
T_0/T_0^*	0	1	∞
fkx/D^*	$-\infty$	0	k

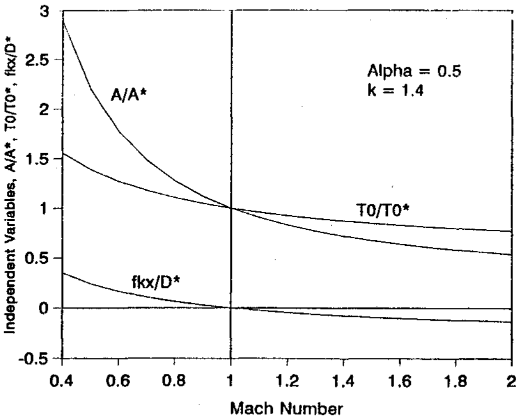

Fig. 1 Independent variable functions.

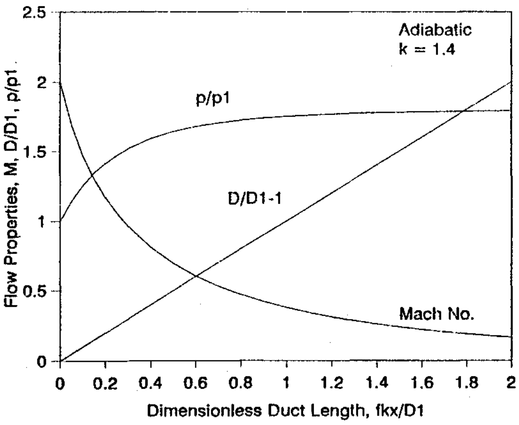

Fig. 2 Flow property relations.

The variations in duct diameter, Mach number, and static pressure are presented in Fig. 2 as a function of the dimensionless duct length. The flow is adiabatic ($\alpha = 1$), and the initial Mach number is 2. The diameter and pressure have been normalized to 1 at the entrance. Figure 2 shows that the duct is a divergent conic. Figure 2 also shows that the transition from a supersonic to a subsonic Mach number is continuous and without a shock.

From the relationship between the duct diameter and length in Fig. 2 and Eq. (2), the divergent angle ϕ can be determined:

$$(D/D_1 - 1)/2x/D_1 = (fkx/D1)/(2\alpha x/D_1) = \tan \phi$$

or

$$f = (2\alpha/k)\tan \phi \qquad (12)$$

For adiabatic flow ($\alpha = 1$) and the typical divergent angle of 15 deg, the value for f is 0.383. This is a relatively high value for the f and is indicative of laminar flow with a Reynolds number in the order of 1×10^2. This range of Reynolds number agrees well with the shockless transitions obtained in Refs. 1 and 2.

Conclusions

The analysis presents a theoretical explanation for shockless transition from supersonic to subsonic that has been observed experimentally and numerically. The analysis shows that shockless transition can only occur with the change in two independent variables. This is realistic since friction is always present. The analysis is based on generalized one-dimensional flow, which is somewhat limiting. Nevertheless, generalized one-dimensional flow results do provide an explanation for anomalies and guidance to more complex studies.

The analysis demonstrates shockless transition by removing the singularity that exists at Mach 1. This results in a set of "Mach functions" for the independent and dependent variables, which describe the direction of the transition. The functions indicate that transition from subsonic to supersonic flow is possible in convergent ducts without a throat under certain conditions.

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Numerical Studies on Droplet Breakup Models

Y. M. Kim*
Hanyang University,
Seoul 133-791, Republic of Korea
H. M. Shang† and C. P. Chen‡
University of Alabama in Huntsville,
Huntsville, Alabama 35899
and
T. S. Wang§
NASA Marshall Space Flight Center,
Huntsville, Alabama 35812

Introduction

ONE of the important aspects in spray combustion modeling is the dense-spray effects that include atomization process, drop breakup, droplet collision, and coalescence. Atomization process occurs on time and length scales too short to be resolved with practical computational grid sizes and time steps. Thus, atomization should be modeled as a sub-grid-scale process. To account for the dense spray effects, the present study employs an existing drop collision and coalescence model[1] and two breakup models, which are the Taylor analogy breakup (TAB) model[2] and the Reitz's wave instability model.[3] In the drop collision model, the probability distributions governing the number and outcomes of the collisions between two drops are sampled randomly in consistency with the stochastic particle tracking method. Both breakup models are based on the assumptions that atomization and drop breakup are indistinguishable processes within a dense spray near the nozzle exit. Accordingly, atomization is prescribed by injecting drops that have a characteristic size equal to the nozzle exit diameter. The present study is mainly motivated to evaluate the performance of these two droplet

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*Associate Professor, Department of Mechanical Engineering. Member AIAA.

†Research Assistant, Department of Chemical Engineering. Member AIAA.

‡Associate Professor, Department of Chemical Engineering. Member AIAA.

§Team Leader, Computational Fluid Dynamics Branch. Member AIAA.